AMPERE's LAW :

The line integral of magnetic field of induction $\vec{B}$ around any closed path in free space is equal to absolute permeability of free space (μ_o) times the total current flowing through area bounded by the path.

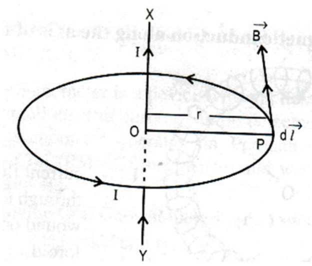
$$\oint \vec{B}.\vec{dl} = \mu_o I$$

In Ampere's law, we imagine an Amperian loop i.e. a closed curve around a current carrying conductor. Further we imagine the loop (selected according to our convenience) to be made up of large number of small elements each of length dl. Then we determine the scalar products of $\vec{B}$ and $\vec{dl}$ for each element and add all such products for entire loop. The direction along which loop is traced is the direction of element of length $\vec{dl}$

$\oint \vec{B}.\vec{dl} = \oint B.dlcos\theta$, where θ is the angle between $\vec{B}$ and $\vec{dl}$

MAGNETIC FIELD DUE TO A LONG STRAIGHT CONDUCTOR CARRYING CURRENT:

Consider as infinitely long straight conductor XY carrying an electric current I. Let P be a point at a distance r from the conductor. We have to determine the magnetic induction of magnetic field at P due to current flowing through the conductor.



Let us choose an Amperian loop as an imaginary
circle of radius r (perpendicular to straight conductor).

$\vec{B}$: magnetic induction at P, due to current I flowing through the conductor.

$\vec{dl}$: length of small element of circle around the wire

According to Ampere's law,

$\oint \vec{B}.\vec{dl} = \mu_o I$, but $\oint \vec{B}.\vec{dl} = \oint B.dlcos\theta$, where θ is the angle between $\vec{B}$ and $\vec{dl}$ which is zero. Thus,

$$\oint B.dl = \mu_o I \qquad Therefore, B\oint dl = \mu_o I$$

$$B(2\pi r) = \mu_o I$$

$$B = \frac{\mu_o I}{2\pi r} = \frac{\mu_o}{4\pi}\frac{2I}{r}$$

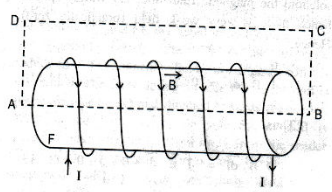
Magnetic Induction along the axis of a long straight solenoid:

Let n: number of turns per unit length

I: current sent through it, due to which magnetic field is created.

Consider a rectangular path ABCD. Let AB=L. Hence, nL is the number of turns enclosed by the rectangle ABCD.

Total current flowing = nLI



$\vec{B}$: Magnetic induction at a point well inside the solenoid.

According to Ampere's law , $\oint \vec{B}.\vec{dl} = \mu_o(nLI). But\ for\ ABCDA,$

$$\oint \vec{B}.\vec{dl} = \oint_A^B \vec{B}.\vec{dl} + \oint_B^C \vec{B}.\vec{dl} + \oint_C^D \vec{B}.\vec{dl} + \oint_D^A \vec{B}.\vec{dl}$$

$$\oint_B^C \vec{B}.\vec{dl} = \oint_D^A \vec{B}.\vec{dl} = 0 \ ,since\ \vec{B}\ is\ perpendicular\ to\ BC\ and\ AD$$

$$\oint_C^D \vec{B}.\vec{dl} = 0,$$

Since outside the solenoid the magnetic field lines are widely spaced hence there is a very weak field (practically zero)

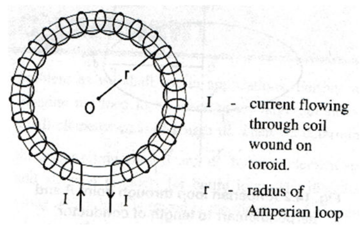
$$Thus, \oint \vec{B}.\vec{dl} = \oint_A^B \vec{B}.\vec{dl} = \oint_A^B Bdlcos\theta = \oint_A^B Bdlcos0 = \oint_A^B Bdl$$

$$B\oint_A^B dl = B.L$$

Thus, $\oint \vec{B}.\vec{dl} = \mu_o(nLI)$ becomes B.L=$\mu_o(nLI)$. Therefore, $B = \mu_o(nI)$

Near the ends, $B = \mu_o(nI)/2$

MAGNETIC INDUCTION ALONG THE AXIS OF TOROID:



Toroid is a solenoid bent into a shape of a hollow donut.

Consider a Amperian loop of radius r.

According to Ampere's law, $\oint \vec{B}.\vec{dl} = \mu_o I$, but here the total current flowing is NI

where N is the total number of turns. Thus, $\oint \vec{B}.\vec{dl} = \mu_o NI$. Now, $\vec{B}$ and $\vec{dl}$ are in same direction. Thus $\oint \vec{B}.\vec{dl} = B.dl = B\,(2\pi r)$

Hence, B(2πr) = $\mu_o NI$ Thus, B= $\frac{\mu_o NI}{2\pi r} = \mu_o nI$

where n = number of turns per unit length of toroid. = $\frac{N}{2\pi r}$